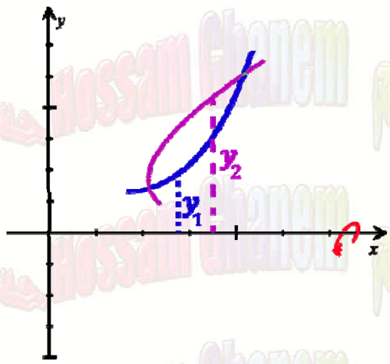
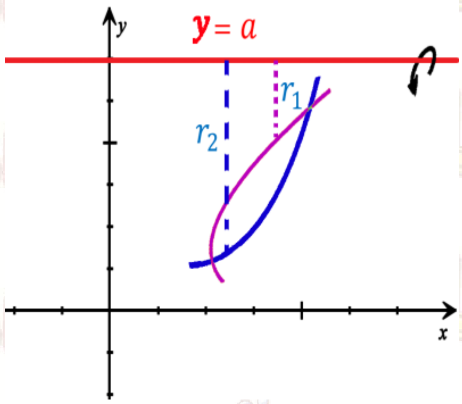
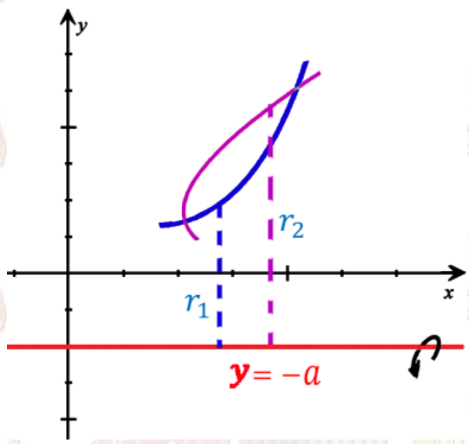
HOSSAM GHANEM

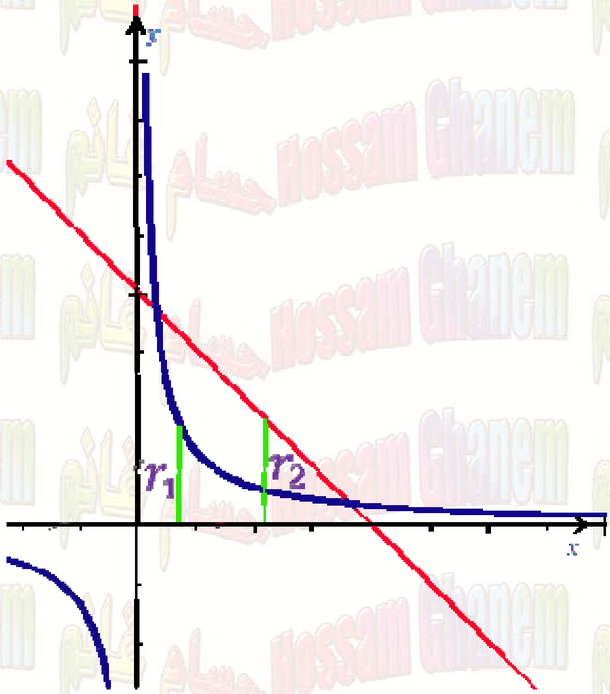
(49) 6.2 Volumes By Washer Method

	$x - axis$
Washer Method	$V = \pi \int_a^b (r_2^2 - r_1^2)dx$

Washer Method $x - axis$ $V = \pi \int_a^b (y_2^2 - y_1^2)dx$	$y_2 = f(x)$ $y_1 = g(x)$	
Washer Method $y = a \quad , \quad a > 0$ $V = \pi \int_a^b (r_2^2 - r_1^2)dx$	$y_2 = f(x)$ $y_1 = g(x)$ $r_2 = a - y_2$ $r_1 = a - y_1$	
Washer Method $y = -a \quad , \quad a > 0$ $V = \pi \int_a^b (r_2^2 - r_1^2)dx$	$y_2 = f(x)$ $y_1 = g(x)$ $r_2 = y_2 + a$ $r_1 = y_1 + a$	

Example 1 14 January 6, 1996	the region in the first quadrant bounded by the graphs of the curves $y = \frac{2}{x}$ and $x + y = 3$ is revolved abut the $x - axis$. Find the volume of resulting solid

Solution



$y = \frac{2}{x}$ & $y = 3 - x$

$$V = \pi \int_1^2 {r_2}^2 - {r_1}^2 \ dx$$

$$V = \pi \int_1^2 (3-x)^2 - \left(\frac{2}{x}\right)^2 \ dx$$

$$= \pi \int_1^2 \left(9 - 6x + x^2 - \frac{4}{x^2}\right) dx$$

$$= \pi \int_1^2 (9 - 6x + x^2 - 4x^{-2}) \ dx$$

$$= \pi \left[\frac{1}{3}x^3 - 3\,x^2 + 9x + 4x^{-1}\right]_1^2$$

$$= \pi \left[\frac{8}{3} - 12 + 18 + 2 - \left(\frac{1}{3} - 3 + 9 + 4\right)\right]$$

$$= \pi \left[\frac{8}{3} + 8 - \left(\frac{1}{3} + 10\right)\right] = \pi \left[\frac{8}{3} - 2 - \frac{1}{3}\right]$$

$$= \pi \cdot \frac{8 - 6 - 1}{3} = \frac{\pi}{3}$$

Example 2 24 May 27. 2001	Set up an integral that can be used to find the volume of the solid obtained by revolving the region bounded by the graphs of the equation $y = 4x - x^2$, and $y = x$ about the line $y = -2$

Solution

Intersection point

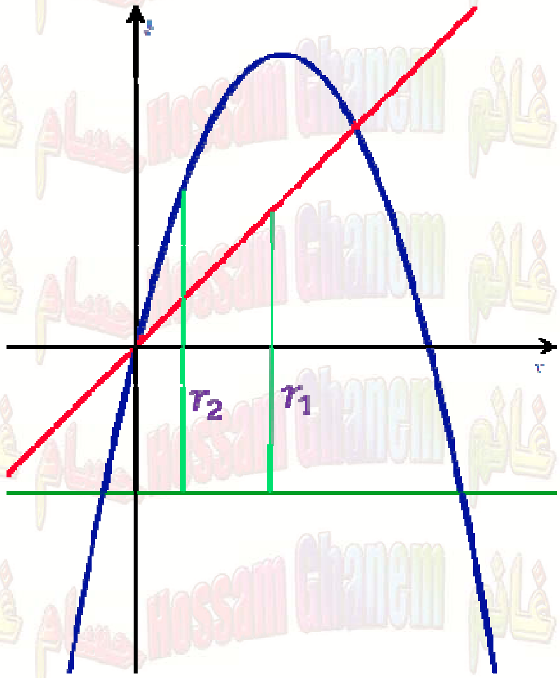
$4x - x^2 = x$

$x^2 - 4x + x = 0$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0$, $x = 3$



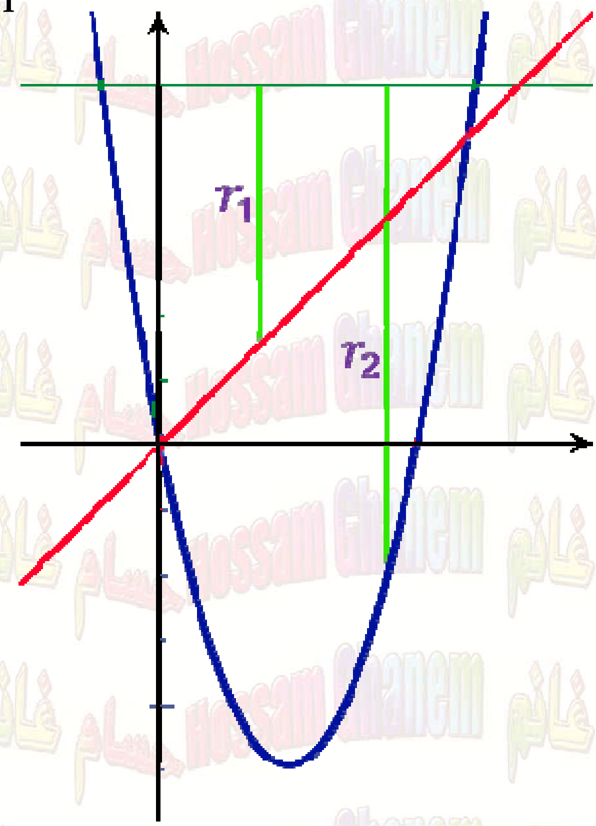
$$V = \pi \int_a^b ({r_2}^2 - {r_1}^2) \ dx$$

$$= \pi \int_0^3 (y_2 + 2)^2 - (y_1 + 2)^2 \ dx$$

$$= \pi \int_0^3 (4x - x^2 + 2)^2 - (x + 2)^2 \ dx$$

Example 3 40 August 7, 2011	(3 Points) Set up an integral for the volume that is obtained by revolving the region enclosed between the curves $y = x^2 - 5x$ and $y = x$ about the lines $y = 7$

Solution



Intersection point

$x^2 - 5x = x$

$x^2 - 6x = 0$

$x(x - 6) = 0$

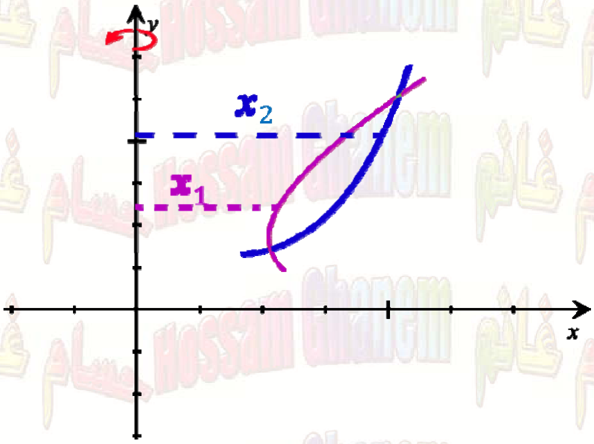
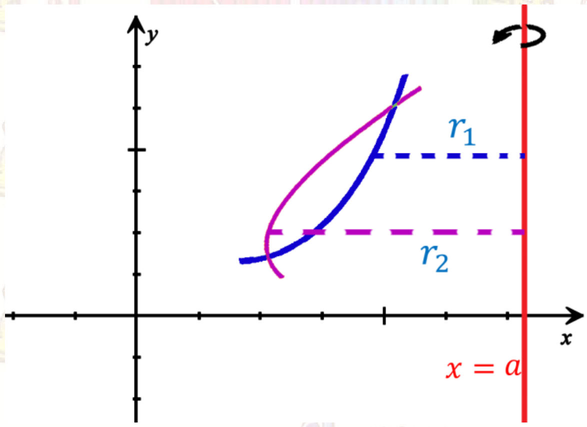
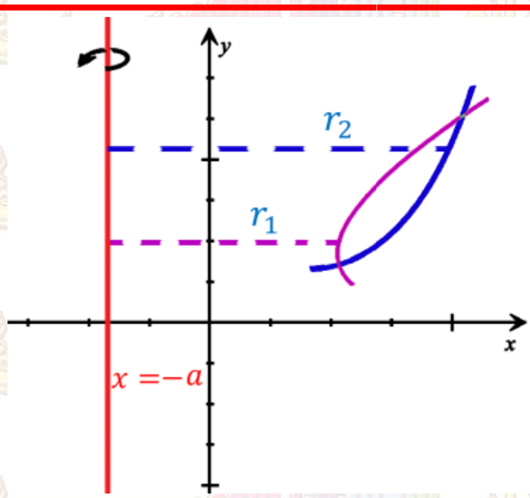
$x = 0 \ , \quad x = 6$

$$V = \pi \int_a^b (r_2{}^2 - r_1{}^2) \ dx$$

$$= \pi \int_0^6 (7 - y_2)^2 - (7 - y_1)^2 \ dx$$

$$= \pi \int_0^3 (7 - x^2 - 5x)^2 - (7 - x)^2 \ dx$$

	$y-axis$
Washer Method	$V=\pi\int_c^d (r_2^2-r_1^2)dy$

Washer Method $y-axis$ $V=\pi\int_a^b (x_2^2-x_1^2)dy$	$x_2=f(y)$ $x_1=g(y)$	
Washer Method $x=a\ ,\ \ a>0$ $V=\pi\int_c^d (r_2^2-r_1^2)dy$	$x_2=f(y)$ $x_1=g(y)$ $r_2=a-x_2$ $r_1=a-x_1$	
Washer Method $x=-a\ ,\ \ a>0$ $V=\pi\int_c^d (r_2^2-r_1^2)dy$	$x_2=f(y)$ $x_1=g(y)$ $r_2=x_2+a$ $r_1=x_1+a$	

Example 4	Set up an integral for the volume of the solid obtained when the region bounded by $x = y^2$ and $x = 4y$ is revolved abut $y - axis$.

Solution

Intersection point

$y^2 = 4y$

$y^2 - 4y = 0$

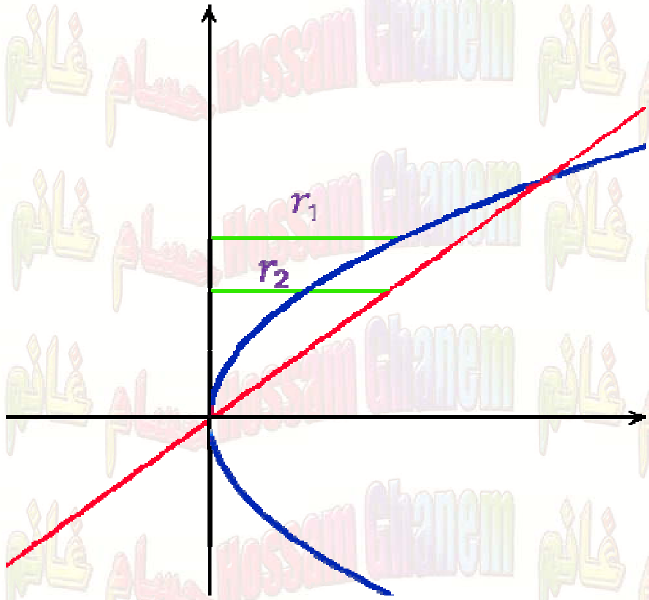
$y(y-4) = 0$

$y = 0 \quad , \quad y = 4$

$$V = \pi \int_a^b ({r_2}^2 - {r_1}^2)\ dy$$

$$= \pi \int_0^4 (4y)^2 - (y^2)^2\ dy$$

$$= \pi \int_0^4 16y^2 - y^4\ dy$$



Example 2 29 June 4. 2007	The region bounded by the curves $x = y^2$ and $x = y^3$ is revolved abut the line $x = 5$. Set up an integral that can be used to find the volume of the resulting

Solution

$y^3 = y^2$

$y^3 - y^2 = 0$

$y^2(y-1) = 0$

$y = 0 \quad , \quad y = 1$

$$V = \pi \int_a^b ({r_2}^2 - {r_1}^2)\ dy$$

$$V = \pi \int_0^1 (5 - x_2)^2 - (5 - x_1)^2\ dy$$

$$V = \pi \int_0^1 (5 - y^3)^2 - (5 - y^2)^2\ dy$$